UNIVERSITY OF KERALA
Model Question Paper
First Degree Programme in Chemistry
Semester IV Complementary Course for Chemistry
MM 1431.2 Mathematics - IV
Abstract Algebra, Linear Transformation and Co-ordinate Systems

Time: 3 hours **Maximum Marks: 80**

Section-I

All the first 10 questions are compulsory. They carry 1 mark each.

1. Give an example of a non-abelian group.
2. Define unit of a ring R.
3. State true or false : The vectors in a basis are linearly independent.
4. Define a dilation from $\mathbb{R}^2$ to $\mathbb{R}^2$.
5. Let A be a 7×5 matrix. What must m and n be in order to define $T: \mathbb{R}^m \to \mathbb{R}^n$ by $T(x) = Ax$
6. Write down the standard matrix corresponding to the transformation of reflection in the line $x_2 = -x_1$.
7. State true or false: If A contains a row or column of zeros, then 0 is an Eigen value of A.
8. The Jacobian corresponding to the transformation from Cartesian system to spherical polar co-ordinate system is ……..
9. If a vector space V has a basis of n vectors, then every basis of V must consists of exactly ..... vectors.
10. Evaluate: $\int_0^{\pi/2} \int_0^{a\sin\theta} r^2 dr d\theta$

Section-II

Answer any 8 questions from among the questions 11 to 22.
These questions carry 2 marks each.

11. Show that every group G with identity e and such that $x * x = e$ for all $x \in G$ is abelian.
12. Compute the subgroups $\langle 3 \rangle$ and $\langle 5 \rangle$ of the group $\langle \mathbb{Z}_6, +_6 \rangle$
13. Define a zero divisor of a ring and give an example of the same.
14. Let $v_1 = [3 \quad 6 \quad 2]^{\mathrm{T}}$, $v_2 = [-1 \quad 0 \quad 1]^{\mathrm{T}}$, $\mathrm{x} = [3 \quad 12 \quad 7]^{\mathrm{T}}$ and $B = \{v_1, v_2\}$. Find the co-ordinate vector $[\mathrm{x}]_B$ of x relative to B.
15. Define a linear transformation and check whether the transformation T is linear if T is defined by: $T(x_1, x_2) = (2x_1 - 3x_2, x_1 + 4, 5x_2)$.
16. Let T be the linear transformation defined by $T(e_1) = (1, 4)$, $T(e_2) = (-2, 9)$ and $T(e_3) = (3, -8)$, where e_1, e_2 and e_3 are columns of the 3×3 identity matrix. Check whether T is one-one or not.

17. Find the dimension of the null space and the column space of:

$$\begin{pmatrix} 1 & 3 & -4 & 2 & -1 & 6 \\ 0 & 0 & 1 & -3 & 7 & 0 \\ 0 & 0 & 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

18. Find the area of the region bounded by the cardioid: $r = 1 - \cos\theta$

19. Express $\int_0^5 \int_0^2 \int_0^{\sqrt{4-y^2}} f(x, y, z) dx\, dy\, dz$ as an equivalent integral in which the z- integration is performed first, the y-integration second and the x-integration last.

20. Evaluate: $\int_0^{\pi/2} \int_0^{\pi/2} \int_0^1 l^3 \sin\varphi \cos\varphi\, dl d\varphi d\theta$

21. Find the volume of the solid enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 9$ using cylindrical co-ordinates.

22. Find the volume of the solid enclosed by the sphere $x^2 + y^2 + z^2 = 4a^2$ and the planes $z = 0$ and $z = 2a$ using spherical co-ordinates.

Section-III

Answer any 6 questions from among the questions 23 to 31.

These questions carry 4 marks each.

23. Define $*$ on the set of positive rational numbers Q^+ by $a * b = \frac{ab}{4}$. Show that $\langle Q^+, * \rangle$ is a group.

24. Describe the group D_3 of symmetries of an equilateral triangle.

25. Check whether {(−1, 1, 2), (2, −3, 1), (10, −14, 0)} is a basis for $\mathbb{R}^3$ over $\mathbb{R}$ or not.

26. Let $T(x_1, x_2) = (3x_1 + x_2, 5x_1 + 7x_2, x_1 + 3x_2)$. Show that T is one-one. Does T map $\mathbb{R}^2$ onto $\mathbb{R}^3$?

27. Let $b_1 = [1 \quad -3]^T$, $b_2 = [-2 \quad 4]^T$, $c_1 = [-7 \quad 9]^T$, $c_2 = [-5 \quad 7]^T$. Consider the bases of $\mathbb{R}^2$ given by $B_1 = \{b_1, b_2\}$ and $B_2 = \{c_1, c_2\}$. Find the change of co-ordinate matrix from B_2 to B_1 and the change of co-ordinate matrix from B_1 to B_2.

28. Let $A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$ and $B = \{b_1, b_2\}$; for $b_1 = [1 \quad 1]^T$, $b_2 = [5 \quad 4]^T$. Define T from $\mathbb{R}^2$ to $\mathbb{R}^2$ by $T(x) = Ax$. Show that b_1 is an Eigen vector of A. Is A diagonalizable?

29. Let $I = \int_0^\infty e^{-x^2} dx$. Show that $I^2 = \int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dxdy$ and hence evaluate I.

30. Evaluate $\iint_R \sin\theta\, dA$ where R is the region in the first quadrant that is outside the circle $r = 2$ and inside the cardioid $r = 2(1 + \cos\theta)$.

31. Find the volume and centroid of the solid G bounded above by $z = \sqrt{25 - x^2 - y^2}$, below by the xy-plane and laterally by the cylinder $x^2 + y^2 = 9$ using cylindrical co-ordinates.

Section-IV

Answer any 2 questions from among the questions 32 to 35.

These questions carry 15 marks each.

32. a. Show that Z_p, where p is a prime, is a field with respect to the operation addition modulo p and multiplication modulo p.

    b. Find four bases for $\mathbb{R}^3$ over $\mathbb{R}$, no two of which have a vector in common.

33. Define T from $\mathbb{R}^2$ to $\mathbb{R}^2$ by $T(x) = Ax$ where $A = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$. Find a basis B for $\mathbb{R}^2$ with the property that $[T]_B$ is diagonal.

34. a. Use a polar double integral to find the area enclosed by the three-petaled rose $r = \sin 3\theta$

    b. Use polar co-ordinates to evaluate: $\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} (x^2 + y^2)^{3/2} \; dy \, dx$

35. a. Find the mass of the solid with density $\delta(x, y, z) = 3 - z$ that is bounded by the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 3$.

    b. Evaluate $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{0}^{\sqrt{4-x^2-y^2}} \sqrt{x^2 + y^2 + z^2} \; dz \, dy \, dx$
