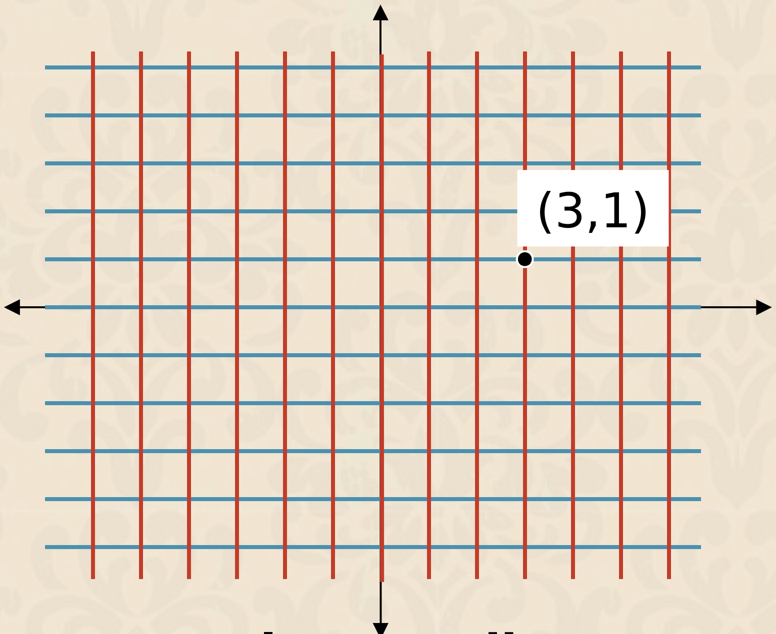
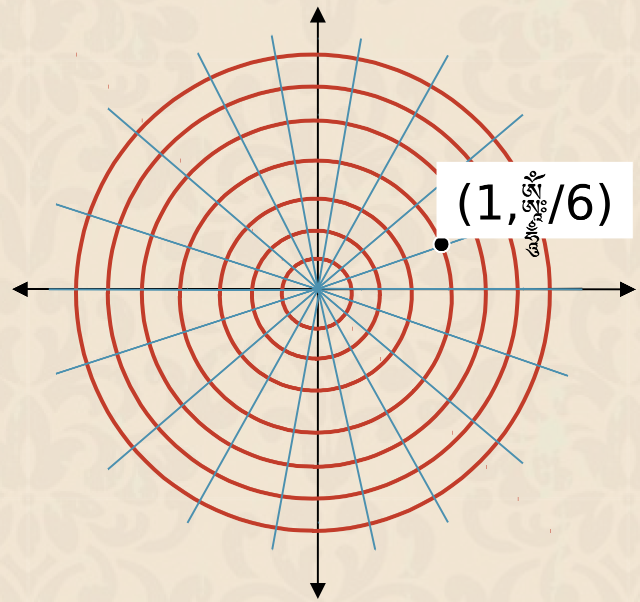
Polar Coordinates

Definition, Plotting Points, Graphs of Polar Equation

Locating Points

Coordinate systems are used to locate the position of a point.





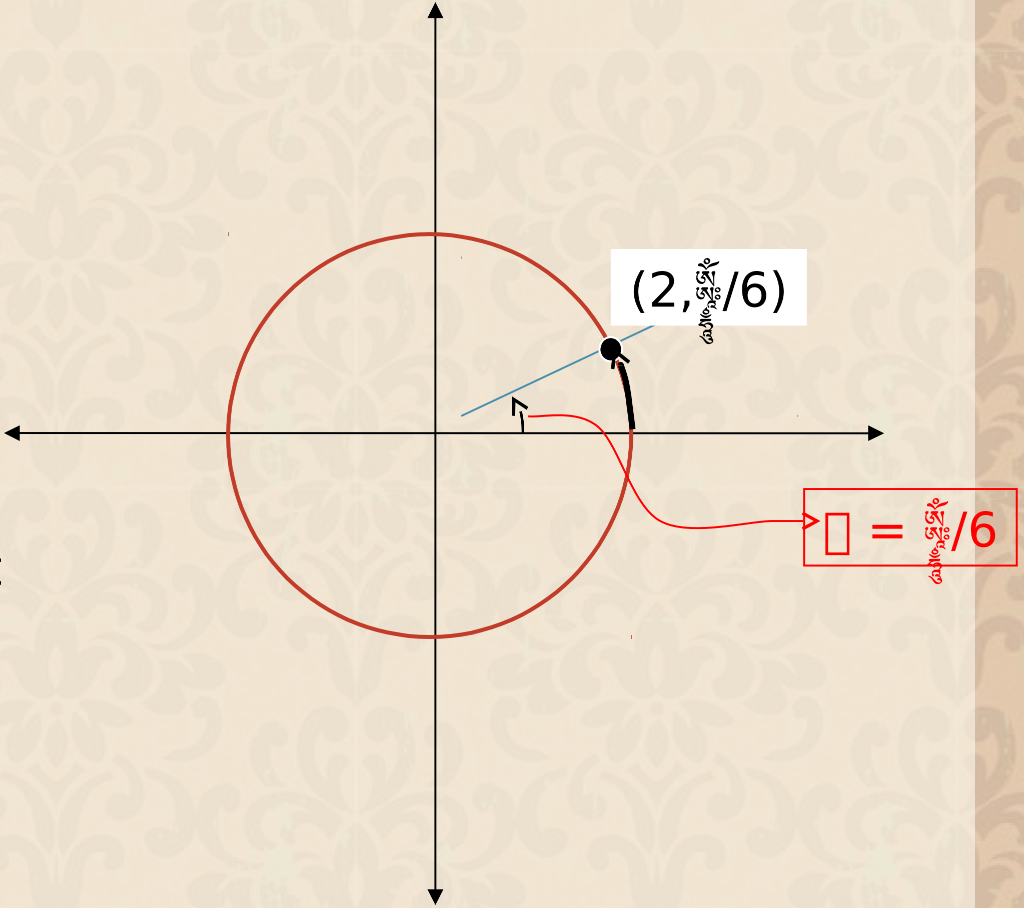
In **rectangular coordinates**:

- We break up the plane into a grid of horizontal and vertical line lines.
- We locate a point by identifying it as the intersection of a vertical and a horizontal line.

In **polar coordinates**:

- We break up the plane with circles centered at the origin and with rays emanating from the origin.
- We locate a point as the intersection of a circle and a ray.

$$(r, \theta) = \left(2, \pi/6 \right)$$

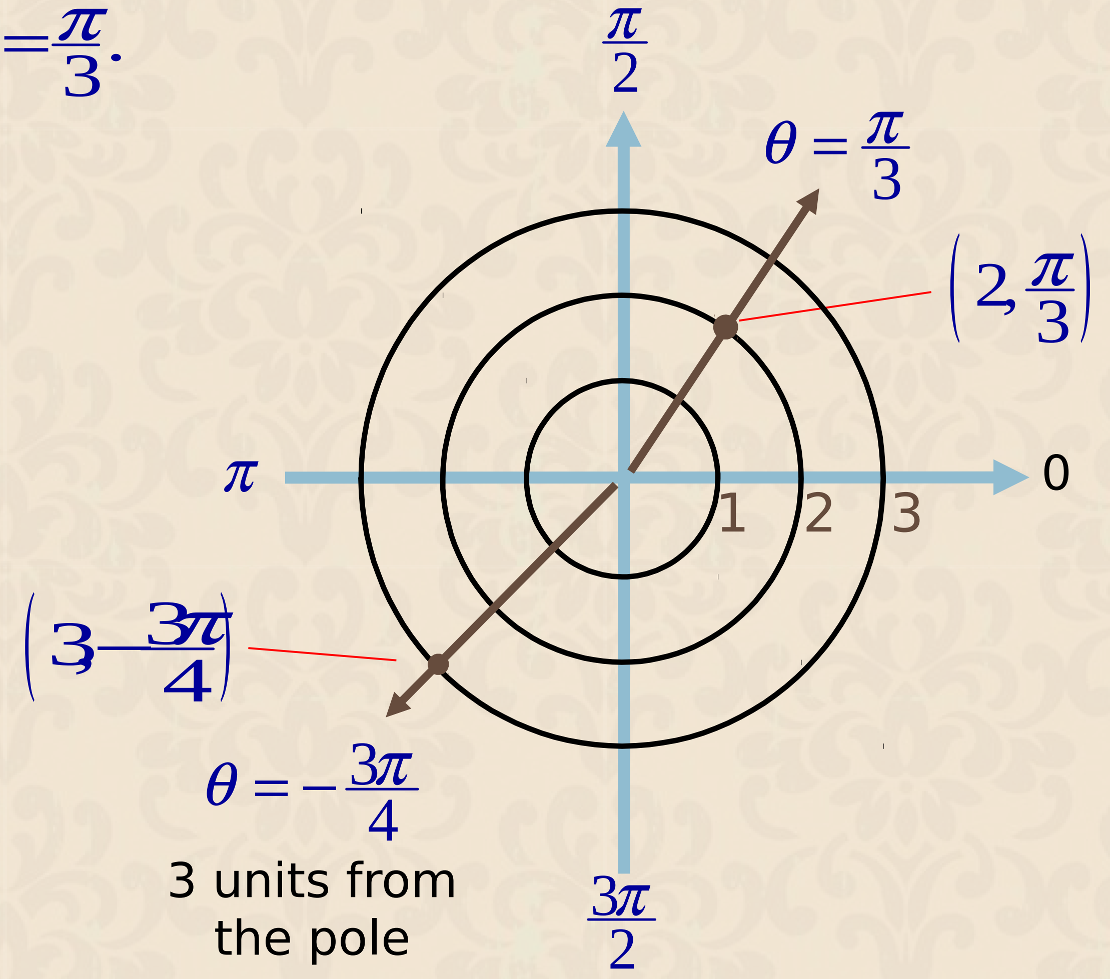


Note, however, that every point in the plane as infinitely many polar representations.

Example 2:

$$(r,\theta)=\left(2,\frac{\pi}{3}\right)$$

$$\theta=\frac{\pi}{3}.$$



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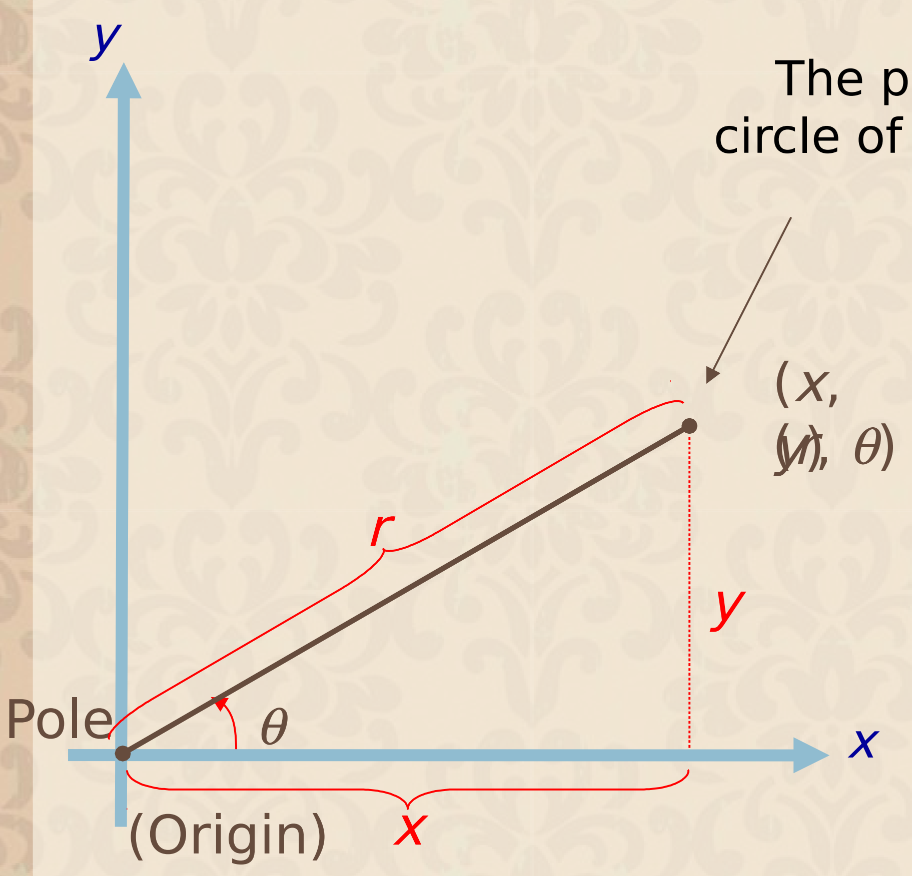
The relationship between rectangular and polar coordinates is as follows.



The point (x, y) lies on a circle of radius r, therefore, $r^2 = x^2 + y^2$.

Definitions of trigonometric functions

$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$

Example: Convert the point (1,1) into polar coordinates.

$$(x, y) = (1, 1)$$

$$\tan\theta = \frac{y}{x} = \frac{1}{1} = 1$$

$$\theta = \frac{\pi}{4}$$

$$r = \sqrt{x^2 + y^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$$

One set of polar coordinates is $(r, \theta) = \left(\sqrt{2}, \frac{\pi}{4}\right)$.

Another set is $(r, \theta) = \left(-\sqrt{2}, \frac{5\pi}{4}\right)$.

Graphs of Polar Equation

Example 1:

$$r = 7$$

We could square both sides

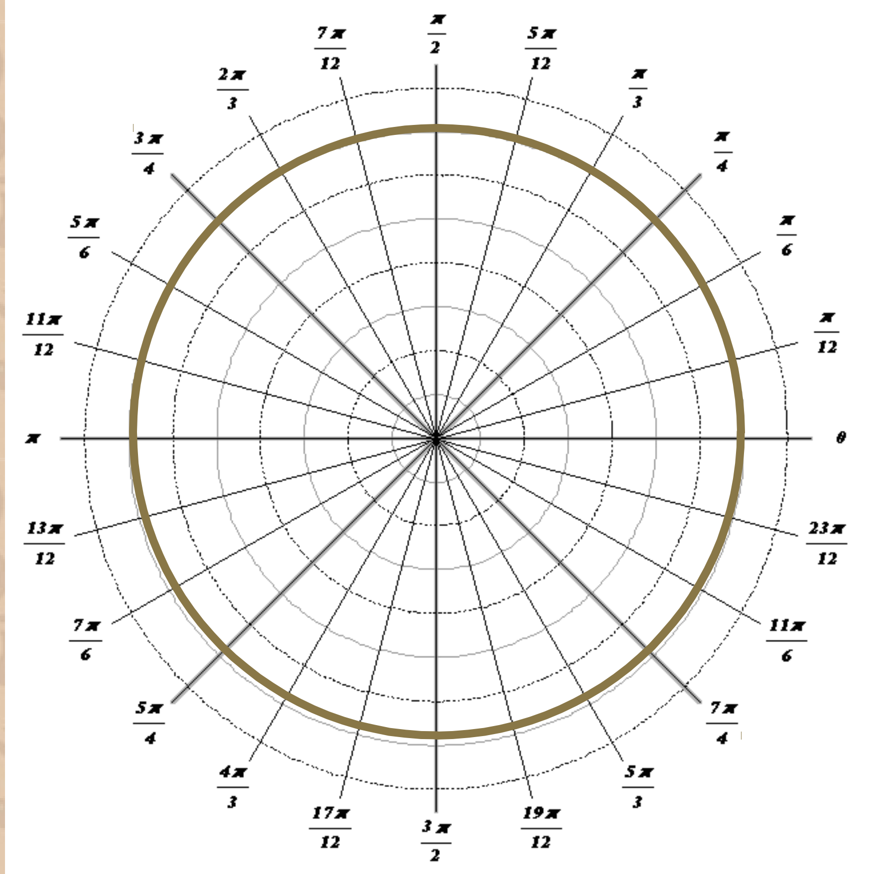
$$r^2 = 49$$

Now use our conversion:

$$r^2 = x^2 + y^2$$

$$x^2 + y^2 = 49$$

We recognize this as a circle with center at (0, 0) and a radius of 7.



On polar graph paper it will centered at the origin and out 7

Exampl,e 2:

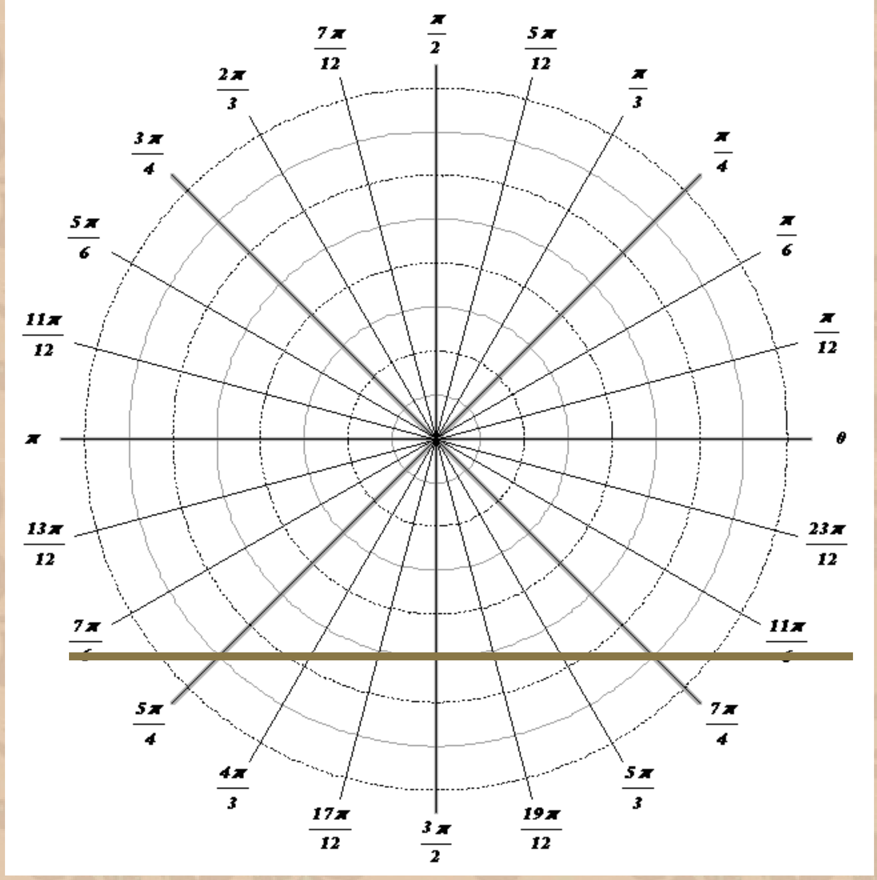
$$r\sin\theta = -5$$

Now use our conversion:

$$y = r\sin\theta$$

$$y = -5$$

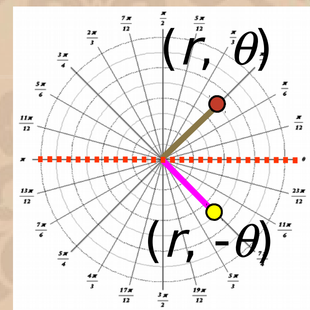
We recognize this as a horizontal line 5 units below the origin (or on a polar plot below the pole)



TESTS FOR SYMMETRY

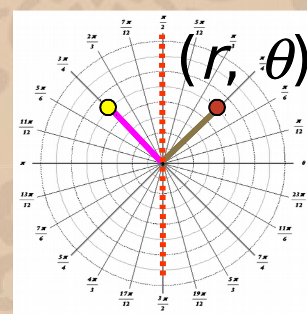
These tests are sufficient but not necessary so if test fails you don't know anything.

Symmetry with Respect to the Polar Axis (*x* axis)



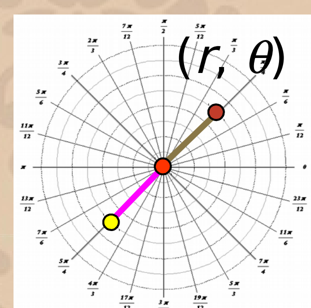
Replace θ by $-\theta$ and if you get original equation back

Symmetry with Respect to the Line $\theta = \pi/2$ (*y* axis)



Replace θ by $\pi - \theta$ and if you get original equation back

Symmetry with Respect to the Pole (Origin)



Replace r by $-r$ and if you get original equation back

Example 1: $r = 1 + 2\cos\theta$

YES!

Let's test for symmetry

Polar Axis: $r = 1 + 2\cos(-\theta)$ $r = 1 + 2\cos\theta$

Line $\theta = \pi/2$: $r = 1 + 2\cos(\pi - \theta)$ **Use the difference formula**

$$r = 1 + 2(\cos\pi\cos\theta + \sin\pi\sin\theta)$$

$r = 1 - 2\cos\theta$

-1 **0**

Not the original equation

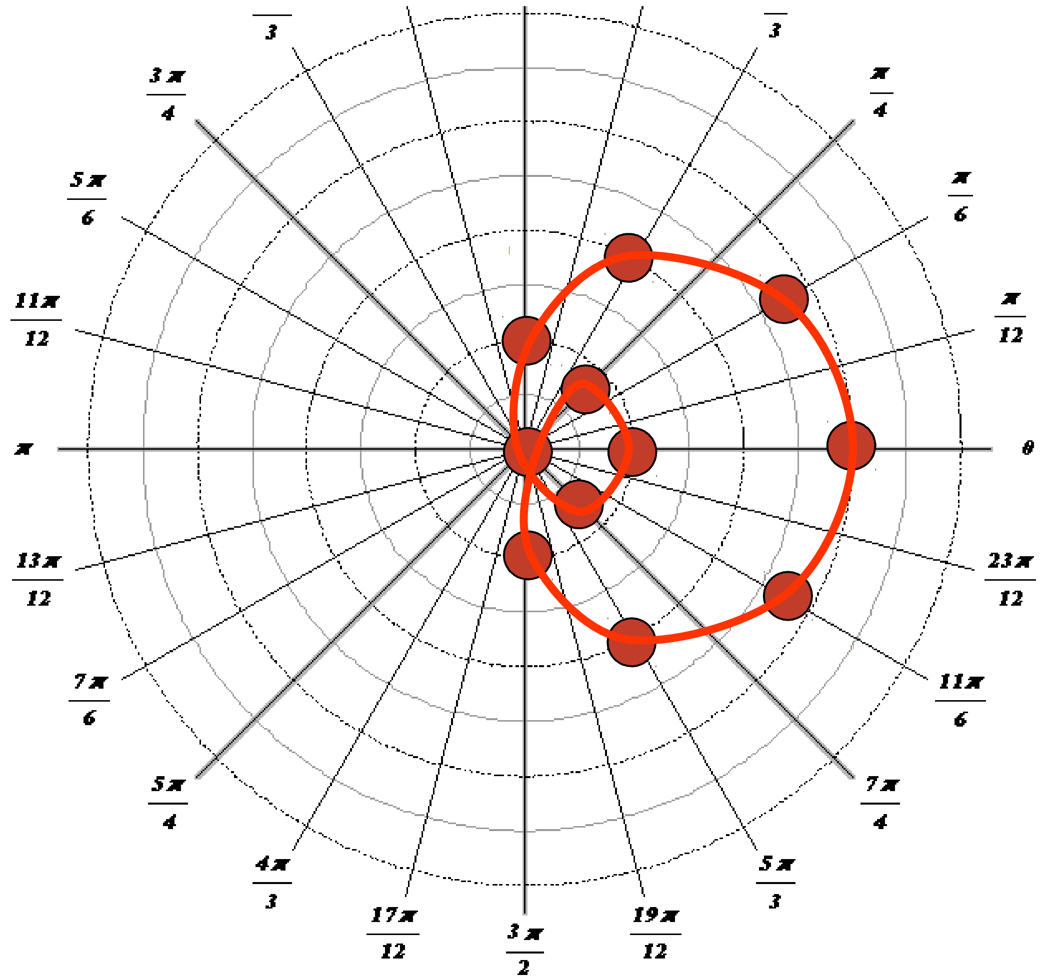
Pole: $-r = 1 + 2\cos\theta$

Not the original equation.

So this graph is symmetric with respect to the polar axis (*x* axis). We will only need to choose θ's on the top half of the graph then and we can use symmetry to get the other half.

θ	$r = 1 + 2\cos\theta$
0	$1 + 2(1) = 3$
$\frac{\pi}{6}$	$1 + 2\left(\frac{\sqrt{3}}{2}\right) \approx 2.73$
$\frac{\pi}{3}$	$1 + 2\left(\frac{1}{2}\right) = 2$
$\frac{\pi}{2}$	$1 + 2(0) = 1$
$\frac{2\pi}{3}$	$1 + 2\left(-\frac{1}{2}\right) = 0$
$\frac{5\pi}{6}$	$1 + 2\left(-\frac{\sqrt{3}}{2}\right) \approx -0.73$
π	$1 + 2(-1) = -1$

This type of graph is called a ***limacon with an inner loop***.



Let's let each unit be 1/2.

Let's plot the symmetric points

Example 2 $$r^2 = 4\sin(2\theta)$$

Let's test for symmetry

Polar Axis: $r^2 = 4\sin(2(-\theta))$ $\quad r^2 = -4\sin(2\theta)$ **FAILS**

Line θ = π/2: $r2 = 4\sin(2(\pi - \theta)) = 4\sin(2\pi - 2\theta)$

sin is periodic so can drop the 2π

$$r^2 = 4\sin(-2\theta) = -2\sin(2\theta)$$ **FAILS**

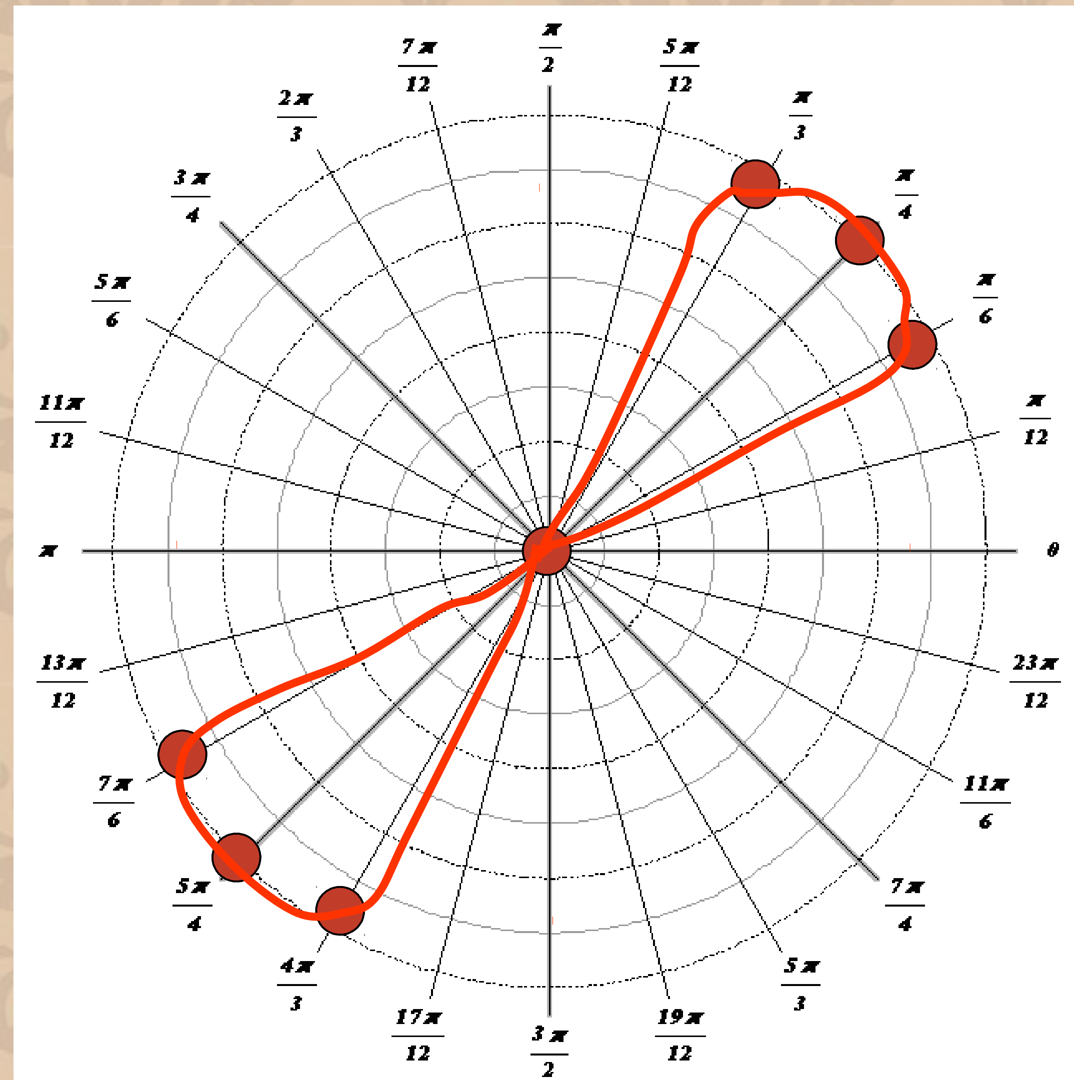
Pole: $(-r)^2 = 4\sin(2\theta)$

$$r^2 = 4\sin(2\theta)$$

So this graph is symmetric with respect to the pole.

This type of graph is called a *lemniscate*

θ	$r^2 = 4\sin(2\theta)$	r
0	$4(0) = 0$	0
$\frac{\pi}{6}$	$4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$	± 1.9
$\frac{\pi}{4}$	$4(1) = 4$	± 2
$\frac{\pi}{3}$	$4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$	± 1.9
$\frac{\pi}{2}$	$4(0) = 0$	0

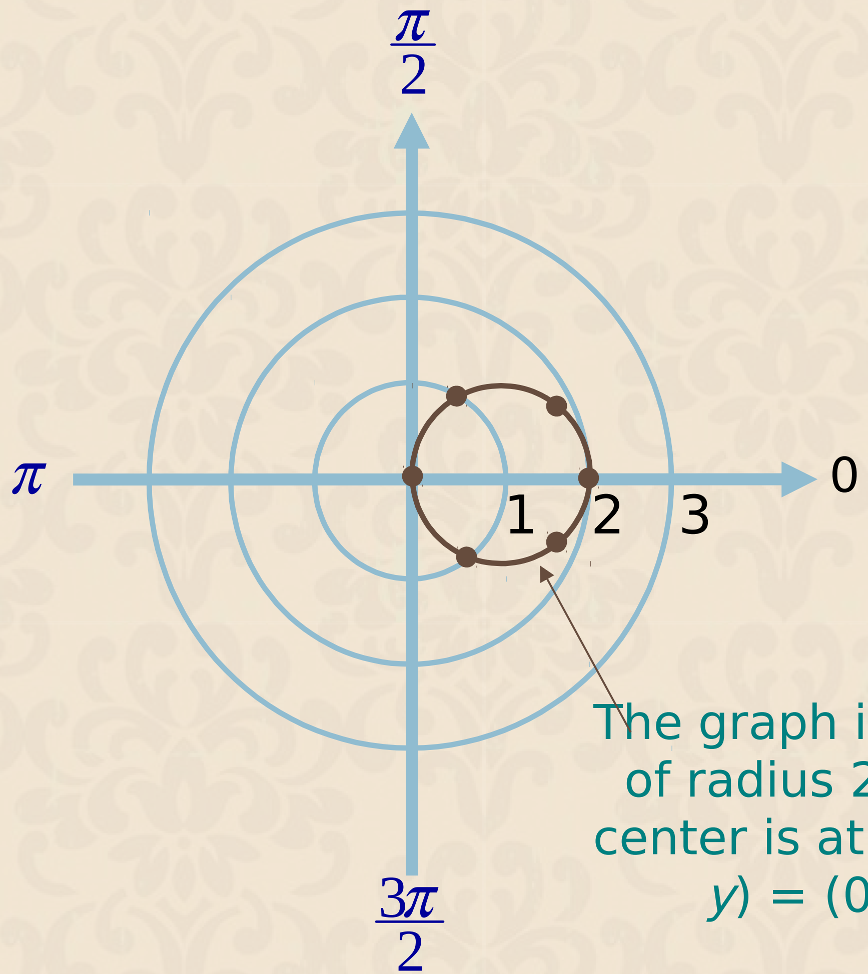


Let's let each unit be 1/4.

Example:3

Graph the polar equation $r = 2\cos\theta$.

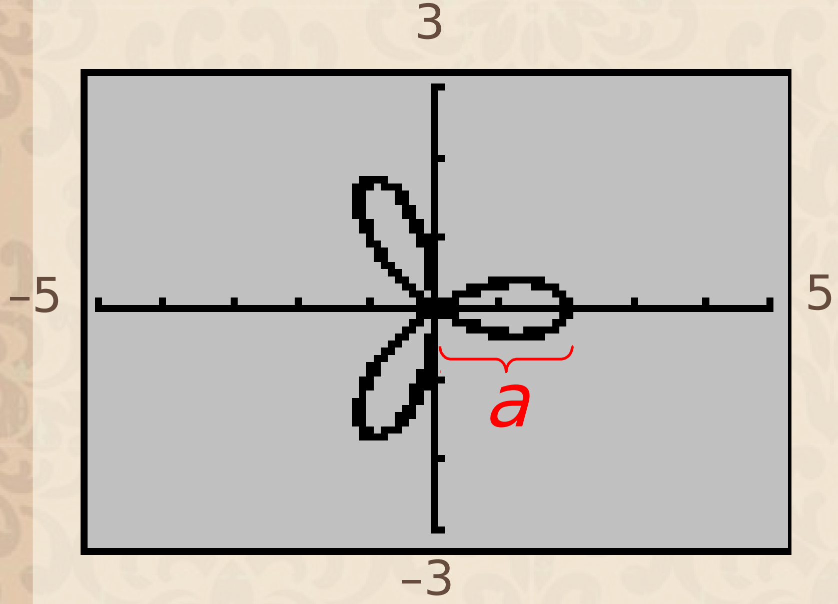
θ	r
0	2
$\frac{\pi}{6}$	$\sqrt{3}$
$\frac{\pi}{3}$	1
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	–1
$\frac{5\pi}{6}$	$-\sqrt{3}$
π	–2
$\frac{7\pi}{6}$	$-\sqrt{3}$
$\frac{3\pi}{2}$	0
$\frac{11\pi}{6}$	$\sqrt{3}$
2π	2

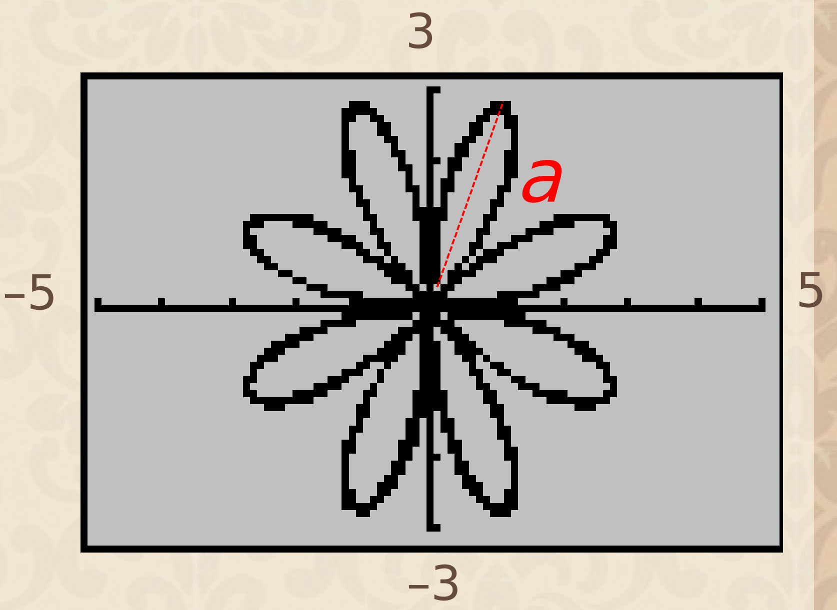


The graph is a circle of radius 2 whose center is at point (x, y) = (0, 1).

Each polar graph below is called a **Rose curve**.

$r = 2\cos 3\theta$ $\qquad$ $r = 3\sin 4\theta$

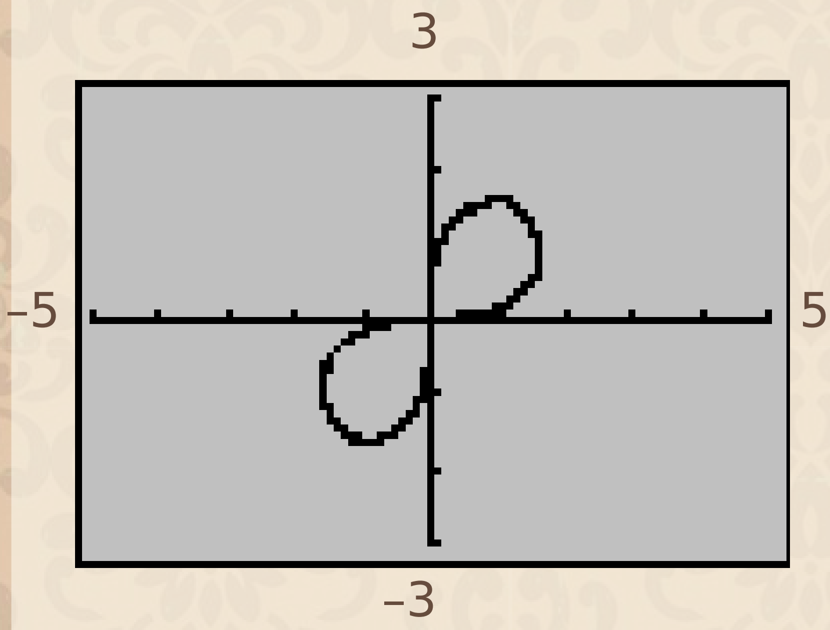




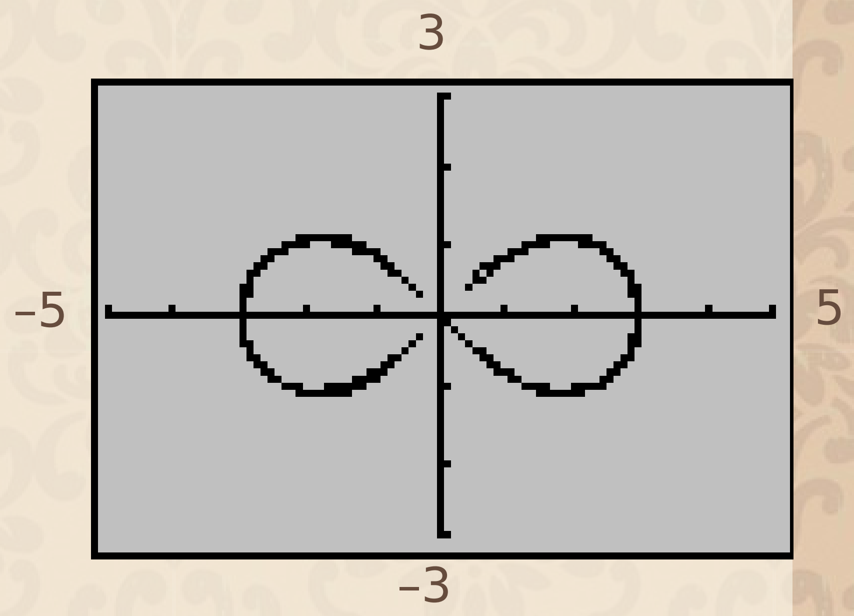
The graph will have *n* petals if *n* is odd, and 2*n* petals if *n* is even.

Each polar graph below is called a **Lemniscate**.

$$r^2 = 2^2 \sin 2\theta$$



$$r^2 = 3^2 \cos 2\theta$$



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